P: ISSN No. 2231-0045 RNI No. UPBIL/2012/55438 VOL.-III, ISSUE-II, November-2014
E: ISSN No. 2349-9443
Periodic Research

Distribution Functions in the Statistical Theory of MHD Turbulence

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Abstract

In this paper we have derived the transport equation for the joint distribution function of velocity and magnetic field. Various properties of constructed distribution functions have been proved

Keywords: Distribution Functions, Hydromagnetic Turbulence, Continuity Equation, Coincidence Property, Reduction Property**.**

Introduction

Two major and distinct areas of investigations in non equilibrium statistical mechanics are the kinetic theory of gases and statistical theory of fluid turbulence. Various analytical theory of turbulence have been given by E. Hopf (1952), R.H. Kraichnan (1969), S. Edward (1964), and J. Herring (1964). Further attempts in this direction were made by T.S. Lundgren (1967). He derived a hierarchy of coupled equations for multipoint turbulent velocity distribution functions which resemble with BBGKY hierarchy of equations in the kinetic theory of gases D. Montgomery (1976) presented a framework for a systematic kinetic theory of inviscid fluid turbulence originating from the Liouville equation for the Fourier coefficients of the fluid variables. Real and imaginary part of these Fourier coefficients play the role in somewhat abstract way, that particle co-ordinates (position and moment) play in the BBGKY theory. This kinetic equation satisfies conservation laws, positive definiteness of spectral densities and H. theorem. Kishore (1977, 1984) constructed and studied distribution functions in the statistical theory of MHD and ordinary turbulence. Pope derived the transport equation for the joint probability density function of velocity and scalars which provide a good basis for modeling turbulent reactive flows. Closure approximations have been presented for the terms involving the fluctuating pressure viscosity and diffusive mixing.

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Dixit (1989,1994,2010) considered the joint distribution function of velocity and Alfven velocity in MHD turbulence. In this paper a hierarchy of distribution functions for simultaneous velocity and magnetic field have been derived. The simple case of one dimensional MHD turbulence has been considered to provide a good basis for the statistical study. Various properties of constructed distribution functions such as reduction, Separation and coincidence have been discussed. The transport equations for one and two point joint distribution functions have been derived and closure has been obtained by a simple relaxation model.

Basic Equation

We start with one dimensional extended Burger's Equation for hydro magnetic turbulence given as (Cf. Kishore and Singh 1984).

$$\partial u/\partial t + u\,\partial u/\partial x - 3h\,\partial h/\partial x - \nu\partial^2 u/\partial x^2 = 0 \qquad (2.1)$$

$$\partial h/\partial t + u\,\partial h/\partial x - h\,\partial u/\partial x - \lambda\partial^2 h/\partial x^2 = 0 \qquad (2.2)$$

With the assumption $\langle u(x,t)\rangle = \langle h(x,t)\rangle = 0$

where u is velocity fluctuation, h magnetic field fluctuation, is kinematics viscosity and is the magnetic diffusivity.

Formulation of the Problem

We consider large identical fluids, each member being and infinite incompressible conducting fluid in turbulent state. No external electric or magnetic field is used to supply the electromagnetic energy in the flow field, but it arises only due to hydro dynamical motion. The fluid and Alfven velocities v and h are randomly distributed functions of position and time and satisfy the equations of motion and continuity given by (2.1) and (2.2). Different members of ensemble are subjected to different initial conditions, and our aim is to find out a way by which we can determine the ensemble averages at the initial time. Certain microscopic properties of conducting

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fluids, such as total energy, total pressure, stress tensor which are nothing but ensemble averages at a particular time, can be determined with the help of the bivariate distribution functions (defined as the averaged distribution functions with the help of Dirac-delta functions. Our present aim is to construct the bivariate distribution functions, study its properties and derive an equation for its evolution.

Bivariate Distribution Functions in Mhd Turbulence and their Properties

Lundgren (1967) has considered the study of flow field on the basis of one variable character only (namely, the fluid velocity v), but we can study it for two or more variables as well. In MHD turbulence we may consider the fluid velocity as well as the Alfven velocity of each point of the flow field. Then, corresponding to each point of the flow field, we have two measurable characteristics. We represent the two variables by v and h and denote the pair of variables at the points $x^{(1)}, x^{(2)}, x^{(3)}........x^{(n)}$ as $\left(v^{(1)}, h^{(1)}\right)$ $\left(v^{(2)}, h^{(2)}..........v^{(n)}, h^{(n)}\right)$ at a fixed instant of time. It is possible that same pair may occur more than once; therefore, we simplify the problem by an assumption that the distribution is discrete (in the sense that no pairs occur more than once). Symbolically we can express the bivariate distribution as

$$\left\{\left(v^{(1)}, h^{(1)}\right), \quad \left(v^{(2)}, h^{(2)}\right) \; .... \; \left(v^{(n)}, h^{(n)}\right)\right\}$$

Instead of considering discrete points in the flow field, if we consider distribution (spatial continuity) of the variables v and h over the entire flow field, statistical behaviour of the fluid may be described by the distribution function F (v,h) which is normalized so that

$$\iint F(v,h)\, dvdh \quad = \quad 1 \tag{4.1}$$

Where the integration ranges over all the possible values of v and h. We shall make use of the same normalization condition for the discrete distributions also. We now define the hierarchy of (v-h) distribution functions in terms of ensemble average. The one point distribution function

$$F_1^{(1)} \quad \left(v^{(1)}, h^{(1)}\right)$$

defined so that

$$F_1^{(1)} \quad \left(v^{(1)}, \; h^{(1)}\right) \; dv^{(1)} dh^{(1)}$$

is the probability that the fluid and Alfven velocities at a time t are in the element $dv^{(1)}$ about $v^{(1)}$ and $dh^{(1)}$ about $h^{(1)}$, is given by

$$F^{(1)}\left(x^{(1)}, v^{(1)}, h^{(1)}, t\right) = \left\langle \delta\left(u^{(1)} - v^{(1)}\right) \; \delta\left(g^{(1)} - h^{(1)}\right)\right\rangle \tag{4.2}$$

Where δ is the direct-delta function defined as

$$\int \delta(u-v)\,dv = \begin{cases} 1 & \text{at the point u=v} \\ 0 & \text{else where} \end{cases}$$

$\delta\left(u^{(1)} - v^{(1)}\right) \delta\left(g^{(1)} - h^{(1)}\right)$ is distribution function for one member of the ensemble and therefore, $\delta_1^{(1)}$ is the average distribution function for one member of the ensemble and therefore, $\delta_1^{(1)}$ is the average distribution function given by.

$$F_2^{(1,2)} = <\delta\left(u^{(1)} - v^{(1)}\right) \; \delta\left(g^{(1)} - h^{(1)}\right) \; \delta\left(u^{(2)} - v^{(2)}\right) x \times \delta\left(y^{(2)} - h^{(2)}\right) > \tag{4.3}$$

where $v^{(1)}$ $h^{(2)}$ are the velocities at the points $x^{(1)}$ and $x^{(2)}$ at time t, etc. Similarly, we can define an infinite number of multi-point bivariate distribution functions $F_3^{(1,2,3)}$, $F_4^{(1,2,3,4)}$ etc.

The distribution functions so constructed possess the following properties.

(i) Reduction Properties

Integration with respect to pair of variables at one point lowers the order of distribution function by one, for example,

$$\iint F_1^{(1)} \; dv^{(1)} \; dh^{(1)} \; = 1$$

$$\iint F_2^{(1,2)} \; dv^{(2)} \; dh^{(2)} \; = F_1^{(1)}$$

$$\iint F_3^{(1,2,3)} \; dv^{(3)} \; dh^{(3)} \; = F_2^{(1,2)}$$

etc. Also the integration with respect to any one of the variables, reduces the number of delta-functions in the distribution function by one, for example,

$$\int F_1^{(1)} \; dv^{(1)} = <\delta\left(g^{(1)} - h^{(1)}\right)>$$

$$\int F_1 \; dh^{(1)} = <\delta\left(u^{(1)} - v^{(v)}\right)>$$

and

$$\int F_2^{(1,2)} \, dh^{(2)} = <\delta\left(u^{(1)} - v^{(1)}\right) \; \delta \; \left(g^{(1)} - h^{(1)}\right) \delta\left(u^{(2)} - v^{(2)}\right)>$$

etc.

(ii) Separation Property

If the two points in the flow field are 'far apart' of each other. The pairs of variables (v,h) at these points should be statistically independent of each other i.e.

$$\lim_{\left|x^{(2)} - x^{(1)}\right| \to \infty} F_2^{(1,2)} = F_1^{(1)} \;\; F_1^{(2)}$$

and similarly,

$$\lim_{\substack{\left|x^{(2)}-x^{(1)}\right|\to\infty \\ \left|x^{(3)}-x^{(2)}\right|\to\infty}} F_3^{(1,2,3)} = F_2^{(1,2)} \;\; F_1^{(3)}$$

(iii) Coincidence Property

When the two points coincide in the flow field,

$$v^{(2)} = v^{(1)} \text{ and } h^{(2)} = h^{(1)}$$

also since

$$\iint F_2^{(1,2)} \, dv^{(2)} \, dh^{(2)} = F_1^{(1)}$$

we have

$$\lim_{x^{(2)}\to x^{(1)}} F_2^{(1,2)} = F_1^{(1)} \, \delta\left(v^{(2)}-v^{(1)}\right) \delta\left(h^{(2)}-h^{(1)}\right)$$

and similarly

$$\lim_{x^{(3)}\to x^{(1)}} F_3^{(1,2,3)} = F_2^{(1,2)} \, \delta\left(v^{(3)}-v^{(1)}\right) \delta\left(h^{(3)}-h^{(1)}\right)$$

etc.

(iv) Symmetry Conditions

$$F_n^{(1,2.......r........s........n)} = F_n^{(1,2.......s...........r..............n)}$$

(v) Incompressibility Conditions

$$(1) \iint \frac{\partial F_n^{(1,2.............n)}}{\partial x^{(r)}} \, v^{(r)} dv^{(r)} \, dh^{(r)} = 0$$

$$(2) \iint \frac{\partial F_n^{(1,2.............n)}}{\partial x^{(r)}} \, h^{(r)} dv^{(r)} \, dh^{(r)} = 0$$

Continuity Equations Expressed in Terms of the Distribution Functions

An infinite number of continuity equations can be derived in the same way as for ordinary turbulence (Hopf, 1952) which will be satisfied if satisfied for initial values of the distributed functions. Taking ensemble averages of equations (2.1) and (2.2), we have

$$0 = <\partial v^{(1)}/\partial x^{(1)}> \; = \; <\partial/\partial x^{(1)} v^{(1)} \iint F_1^{(1)} dv^{(1)} dh^{(1)}>$$

$$= \partial/\partial x^{(1)} <v^{(1)} \iint F_1^{(1)} dv^{(1)} dh^{(1)}> \; = \partial/\partial x \iint v^{(1)} F_1^{(1)} dv^{(1)} dh^{(1)}$$

$$= \partial/\partial x^{(1)} \iint dv^{(1)} F_1^{(1)} dv^{(1)} dh^{(1)} \; = \iint \partial F^{(1)}/\partial x^{(1)} v^{(1)} dv^{(1)} dh^{(1)}$$

and similarly

$$0 = \iint \partial F_1^{(1)}/\partial x^{(1)} h^{(1)} dv^{(1)} dh^{(1)};$$

which are the first order continuity equation in which only one point distribution is involved. In a similar way, second order continuity equations can be derived and are found to be

$$\partial/\partial x^{(1)} \iint h^{(1)} \; F_2^{(1,2)} \; dv^{(1)} \; dh^{(1)} = 0$$

$$\partial/\partial x^{(1)} \iint v^{(1)} \; F_2^{(1,2)} \; dv^{(1)} \; dh^{(1)} = 0$$

and the nth order Continuity equations are

$$\partial/\partial x^{(1)} \iint v^{(1)} \; F_n^{(1,2.........n)} \; dv^{(1)} \; dh^{(1)} = 0$$

and

$$\partial/\partial x^{(1)} \iint h^{(1)} \; F_n^{(1,2.........n)} \; dv^{(1)} \; dh^{(1)} = 0$$

the continuity equations are symmetric in their arguments i.e.

$$\partial/\partial x^{(r)} \iint h^{(r)} \; F_n^{(1,2......r......s.....n)} \; dv^{(r)} \; dh^{(r)} = \partial/\partial x^{(s)} \iint h^{(S)} \; F_n^{(1,2.........s.........r.........n)} \; dv^{(s)} \; dh^{(s)}$$

Equations for the Evolution of Joint Distribution Functions

The time evolution of

$F_1^{(1)} = <\delta\left(u^{(1)}-v^{(1)}\right) \delta\left(h^{(1)}-g^{(1)}\right)>$ is given by

$$\partial/\partial t \;\; F_1^{(1)} = \partial/\partial t <\delta\left(u^{(1)}-v^{(1)}\right) \delta\left(h^{(1)}-g^{(1)}\right)>$$

$$= \partial/\partial t <[\, \delta\left(u^{(1)}-v^{(1)}\right) \delta\left(h^{(1)}-g^{(1)}\right)]>$$

$$= <\partial/\partial t \;\; \delta\left(u^{(1)}-v^{(1)}\right) \delta\left(h^{(1)}-g^{(1)}\right)>$$

$$+ <\delta\left(u^{(1)}-v^{(1)}\right) \partial/\partial t \;\; \delta\left(h^{(1)}-g^{(1)}\right)>$$

$$= <\left\{\delta u^{(1)}/\partial t \; \partial/\partial v \;\; \delta\left(u^{(1)}-v^{(1)}\right)\right\} \delta\left(h^{(1)}-g^{(1)}\right)>$$

$$+ <-\delta\left(u^{(1)}-v^{(1)}\right) \partial h^{(1)}/\partial t \; \partial/\partial g^{(1)} \; \delta\left(h^{(1)}-g^{(1)}\right)>$$

$$= <-\delta\left(h^{(1)}-g^{(1)}\right) \left\{-u^{(1)} \; \partial u^{(1)}/\partial x^{(1)} + 3h^{(1)} \; \partial h^{(1)}/\partial x^{(1)}\right.$$

$$\left. + \;\; v\partial/\partial x^{(1)} \; \partial/\partial x^{(1)} \; u^{(1)}. \; \partial/\partial v^{(1)} \delta\left(u^{(1)}-v^{(1)}\right)\right\}>$$

$$+ <-\delta\left(u^{(1)}-v^{(1)}\right) \qquad \left\{-u^{(1)} \partial h^{(1)}/\partial x^{(1)}\right.$$

$$+ h^{(1)} \; \partial u^{(1)}/\partial x^{(1)}$$

$$\left. + \;\; \lambda\partial/\partial x^{(1)} \; \partial/\partial x^{(1)} \; h^{(1)}. \; \partial/\partial g^{(1)} \delta\left(h^{(1)}-g^{(1)}\right)\right\}> \tag{6.1}$$

$$= <u^{(1)} \; \delta\left(h^{(1)}-g^{(1)}\right) \partial u^{(1)}/\partial x^{(1)} \; \partial/\partial v^{(1)} \; \delta\left(u^{(1)}-v^{(1)}\right)>$$

$$+ \; 3<-\delta\left(h^{(1)}-g^{(1)}\right) h^{(1)} \partial h^{(1)}/\partial x^{(1)} \;\; \partial/\partial v^{(1)} \delta\left(u^{(1)}-v^{(1)}\right)>$$

$$- \; <v \; \delta\left(h^{(1)}-g^{(1)}\right) \partial/\partial x^{(1)} \partial/\partial x^{(1)} \; u^{(1)} \; \partial/\partial v^{(1)} \delta\left(u^{(1)}-v^{(1)}\right)>$$

$$+ \; <\delta\left(u^{(1)}-v^{(1)}\right) u^{(1)} \; \partial h^{(1)}/\partial x^{(1)} \;\; \partial/\partial g^{(1)} \delta\left(h^{(1)}-g^{(1)}\right)>$$

$$+ \; <-\delta\left(u^{(1)}-v^{(1)}\right) h^{(1)} \; \partial u^{(1)}/\partial x^{(1)} \;\; \partial/\partial g^{(1)} \delta\left(h^{(1)}-g^{(1)}\right)>$$

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$$+ \lambda < -\delta\left(u^{(1)}-v^{(1)}\right)\left\{\partial/\partial x^{(1)}\ \partial/\partial x^{(1)} h^{(1)}\right\} \partial/\partial g^{(1)} \delta\left(h^{(1)}-g^{(1)}\right)> \quad (6.2)$$

By calculation

$$= \ < u^{(1)}\delta\left(h^{(1)}-g^{(1)}\right) \partial u^{(1)}/\partial x^{(1)}\ \partial/\partial v^{(1)}\delta\left(u^{(1)}-v^{(1)}\right)>$$

$$= \ - < u^{(1)}\delta\left(h^{(1)}-g^{(1)}\right) \partial/\partial x^{(1)}\ \delta\left(u^{(1)}-v^{(1)}\right)> \quad (6.3)$$

$$< \delta\left(u^{(1)}-v^{(1)}\right) u^{(1)}\partial h^{(1)}\ /\partial x^{(1)}\ \partial/\partial g^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)>$$

$$= \ < \delta\left(u^{(1)}-v^{(1)}\right) u^{(1)}\partial/\partial x^{(1)}\ \delta\left(h^{(1)}-g^{(1)}\right)> \quad (6.4)$$

Adding (6.3) and (6.4) we have

$$-u^{(1)}\ \partial/\partial x^{(1)}\ F_1^{(1)} \quad (6.5)$$

$$< -\nu\delta\left(h^{(1)}-g^{(1)}\right)\partial/\partial v^{(1)}\ \delta\left(u^{(1)}-v^{(1)}\right)\partial/\partial x^{(1)}\ \partial/\partial x^{(1)} u^{(1)}>$$

$$= \ - \ \partial/\partial u^{(1)} \lim_{x^{(2)}\to x^{(1)}} \partial/\partial x^{(2)}\ \partial/\partial x^{(2)} \int u^{(1)} F_2^{(1,2)} du^{(2)} dh^{(2)} \quad (6.6)$$

$$\lambda < -\delta\left(u^{(1)}-v^{(1)}\right) \partial/\partial g^{(1)}\ \delta\left(h^{(1)}-g^{(1)}\right)\partial/\mu x^{(1)}\ \partial/\partial x^{(1)} h^{(1)}>$$

$$= \ -\lambda\partial/\partial\ h^{(1)} \lim_{x^{(2)}\to x^{(1)}} \partial/\partial x^{(1)}\ \partial/\partial x^{(2)} \int h^{(2)}\ F_2^{(1,2)} du^{(2)} dh^{(2)} \quad (6.7)$$

$$3 < -\delta\left(h^{(1)}-g^{(1)}\right) h^{(1)}\partial h^{(1)}/\partial x^{(1)}\partial/\partial v^{(1)}\ \delta\left(u^{(1)}-u^{(1)}\right)>$$

$$= \ -3h^{(1)}\ \partial h^{(1)}/\partial u^{(1)}\ \partial/\partial x^{(1)}\ F_1^{(1)} \quad (6.8)$$

and

$$< -\delta\left(u^{(1)}-v^{(1)}\right) h\,\partial u^{(1)}/\partial x^{(1)}\ \partial/\partial g^{(1)}\delta\left(h^{(1)}-g^{(1)}\right)>$$

$$= \ - \ h^{(1)}\ \partial u^{(1)}/\partial h^{(1)}\partial/\partial x^{(1)}\ F_1^{(1)} \quad (6.9)$$

Putting equations (6.3) ----------- (6.9) in equation (6.2) we have

$$\partial F^{(1)}/\partial t + u^{(1)}\ \partial F^{(1)}/\partial x^{(1)} + 3h^{(1)}\partial h^{(1)}/\partial u^{(1)}\partial/\partial x^{(1)} F_1^{(1)}$$
$$+ \ \partial/\partial u^{(1)} \lim_{x^{(2)}\to x^{(1)}} \nu\partial/\partial x^{(2)}\ \partial/\partial x^{(2)} \int u^{(1)} F_2^{(1,2)} du^{(2)} dh^{(2)}$$
$$+ h^{(1)}\partial u^{(1)}/\partial\ h^{(1)}\ \partial/\partial x^{(1)}\ F_1^{(1)} + \lambda\partial/\partial h^{(1)} \lim_{x^{(2)}\to x^{(1)}} \partial/\partial x^{(1)}$$
$$\partial/\partial u^{(2)} \int h^{(2)} F_2^{(1,2)} du^{(2)} dh^{(2)} = 0 \quad (6.10)$$

Similarly a transport equation for two point joint distribution function F can be derived as

$$\partial F_2^{(1,2)}/t + \left(v^{(1)}\partial/\partial x^{(1)} + v^{(2)}\partial/\partial x^{(2)}\right) F_2^{(1,2)}$$
$$+3h^{(1)}\partial h^{(1)}\partial/\partial v^{(1)}\partial/\partial x^{(1)}\ F_2^{(1,2)}$$
$$+3h^{(2)}\partial h^{(2)}/\partial v^{(2)}\partial/\partial x^{(2)}\ F_2^{(1,3)} + h^{(1)}\partial v^{(1)}/\partial h^{(1)}$$
$$\partial/\partial x^{(1)}\ F_2^{(1,2)}$$
$$+h^{(2)}\partial v^{(2)}/\partial h^{(2)}\partial/\partial x^{(2)}\ F_2^{(1,3)} + \nu(\partial/\partial v^{(1)} \lim_{x^{(3)}\to x^{(1)}} + \partial/\partial v^{(2)} \lim_{x^{(3)}\to x^{(2)}})$$
$$\partial/\partial x^{(3)}\partial/\partial x^{(3)} \int v^{(3)} F_3^{(1,2,3)} dv^{(3)} dh^{(3)} + \lambda(\partial/\partial h^{(1)} \lim_{x^{(3)}\to x^{(1)}} +$$
$$\partial/\partial h^{(2)} \lim_{x^{(3)}\to x^{(1)}})\ \partial/\partial x^{(2)} \int h^{(2)} F_2 du^{(2)} dh^{(2)} = 0 \quad (6.11)$$

Closure Scheme and Discussion

In order to close the Transport Equations for the joint distribution functions, some approximations are required. Here closer is obtained by

$$F_2^{(1,2)} = \left(1+\theta\right)\ F_1^{(1)}\ \ F_1^{(2)} \quad (7.1)$$

and

$$F_3^{(1,2,3)} = \left(1+\theta\right)^2\ F_1^{(1)}\ \ F_1^{(2)}\ \ F_1^{(3)} \quad (7.2)$$

Where θ is correlation coefficient.

When θ = 0. That is the case in which magnetic diffusivity is so small as to be negligible in comparison to kinematic viscosity and in this case instability to small magnetic perturbation is to be expected. The relevant equations are.

$$\partial F^{(1)}/\partial t + u^{(1)}\partial F^{(1)}/\partial x^{(1)} + 3h^{(1)}\partial h^{(1)}/\partial u^{(1)}\partial/\partial x^{(1)} F_1^{(1)}$$
$$+ \ \partial/\partial u^{(1)} \lim_{x^{(2)}\to x^{(1)}} \nu\ \partial/\partial x^{(2)}\ \partial/\partial x^{(2)}$$
$$\int u^{(1)} F_2^{(1,2)} du^{(2)} dh^{(2)} + h^{(1)}\partial u^{(1)}/\partial h^{(1)}\partial/\partial x^{(1)} = F_1^{(1)} = 0$$

(7.3)

and

$$\partial F_2^{(1,2)}/\partial t + (v^{(1)}\partial/\partial x^{(1)} + v^{(2)}\partial/\partial x^{(2)})\ F_1^{(1,2)} +$$
$$3h^{(1)}\partial h^{(1)}/\partial v^{(1)}\partial/\partial x^{(1)}\ F_2^{(1,2)} + 3h^{(2)}\partial h^{(2)}/\partial v^{(2)}$$
$$\partial/\partial x^{(2)} F_2^{(1,3)} + h^{(1)}\partial v^{(1)}/\partial h^{(1)}\partial/\partial x^{(1)} F_2^{(1,2)} +$$
$$+ h^{(2)}\partial v^{(2)}/\partial h^{(2)}\partial/\partial x^{(2)} F_2^{(1,3)} + \ \nu(\partial/\partial v^{(1)} \lim_{x^{(3)}\to x^{(1)}} +$$
$$+\partial/\partial v^{(2)} \lim_{x^{(3)}\to x^{(2)}})\ \partial/\partial x^{(3)}\partial/\partial x^{(3)} \int v^{(3)} F_3^{(1,2,3)} dv^{(3)} dh^{(3)} \quad (7.4)$$

In weakly turbulent medium, the case when magnetic diffusivity equals the kinematics viscosity turns out to be interesting because in most of the useful fluids electrical conductivity is not very high and in this case the relevant equations are

$$\partial F_1/\partial t\ +\ u^{(1)}\partial F^{(1)}/\partial x^{(1)} 3h^{(1)} \frac{\partial h^{(1)}}{\partial u^{(1)}}\ \partial/\partial x^{(1)} F_1^{(1)}$$
$$+ \partial/\partial u^{(1)} \lim_{x^{(2)}\to x^{(1)}} \nu\ \partial/\partial x^{(2)}\ \partial/\partial x^{(2)} u^{(1)}\ F_2^{(1,2)} du^{(2)} dh^{(2)} +$$
$$+ \ h^{(1)}\ \ \partial u^{(1)}/\partial h^{(1)}\ \partial/\partial x^{(1)}\ F_2^{(1,2)} + \nu\ \partial/\partial h^{(1)} \lim_{x^{(2)}\to x^{(1)}} \partial/\partial x^{(1)}$$
$$\partial/\partial x^{(2)}\ h^{(2)}\ F_2^{(1,2)} du^{(2)} dh^{(2)} = 0 \quad (7.5)$$

and

P: ISSN No. 2231-0045 RNI No. UPBIL/2012/55438 VOL.-III, ISSUE-II, November-2014
E: ISSN No. 2349-9443
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$$\partial F_2^{(1,2)}/\partial t + (\nu^{(1)}\ \partial/\partial x^{(1)} + v^{(2)}\ \partial/\partial x^{(2)})\ F_2^{(1,2)} +$$
$$+ 3h^{(1)}\partial h^{(1)}/\partial v^{(1)}\ \partial/\partial x^{(1)}\ F_2^{(1,2)} + 3h^{(2)}\partial h^{(2)}/\partial v^{(2)}\partial/\partial x^{(2)} F_2^{(1,3)} +$$
$$+ h^{(1)}\partial v^{(1)}/\partial h^{(1)}\ \partial/\partial x^{(1)}\ F_2^{(1,2)} + h^{(2)}\partial v^{(2)}/\partial h^{(2)}\partial/\partial x^{(2)} F_2^{(1,3)}$$
$$+ \nu\ [(\partial/\partial v^{(1)} \lim_{x^{(3)}\to x^{(1)}} + \partial/\partial v^{(2)} \lim_{x^{(3)}\to x^{(2)}})\partial/\partial x^{(3)}\ \partial/\partial x^{(3)} \int v^{(3)}$$
$$F_3^{(1,2,3)} dv^{(3)} dh^{(3)} + (\partial/\partial h^{(1)} \lim_{x^{(3)}\to x^{(2)}} + \partial/\partial h^{(2)} \lim_{x^{(3)}\to x^{(2)}})$$
$$\partial/\partial x^{(2)} \int h^{(2)}\ F_2\ du^{(2)}\ dh^{(2)}\ ] = 0 \qquad (7.6)$$

In order to close the transport equation for the joint bivariate distribution functions approximations are required. If we consider the collection of ionized particles that is in plasma turbulence case, it can be provided closure form easily by decomposing $F_2^{(1,2)}$ as $F_1^{(1)}\ F_1^{(2)}$. But such type of approximations can be possible when there is no interaction or correlation between two particles. We decompose $F_2^{(1,2)}$ as.

$$F_2^{(1,2)} = F_1^{(1)}\ \ F_1^{(1)} + \theta\ F_1^{(2)}\ F_1^{(2)}$$

and

$$F_3^{(1,2,3)} = (1+\theta)^2\ \ F_1^{(1)}\ F_1^{(2)}\ F_1^{(3)}$$

Here θ is correlation coefficient between the particles. If there is no correlation between two particles θ will be zero and distribution function can be decomposed in usual way. Here we are considering such type of approximations only to provide closed form to the equations i.e. to approximate two point equations as one point equations. F (v,h) contains all the statistical information about the velocity at each point, therefore a turbulence model to determine the Reynolds stresses is not needed. Since F(v,h) is one point statistics, the length scale information is also not needed.

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